

Family unification, exotic states and light magnetic monopoles

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ABSTRACT: Models with fermions in bifundamental representations can lead naturally to family unification as opposed to family replication. Such models typically predict (exotic) color singlet states with fractional electric charge, and magnetic monopoles with multiple Dirac charge. The exotics may be at the TeV scale, and relatively light magnetic monopoles ($\gtrsim 10^7$ GeV) can be present in the galaxy with abundance near the Parker bound. We focus on three family $SU(4) \times SU(3) \times SU(3)$ models.

KEYWORDS: Beyond Standard Model, Solitons Monopoles and Instantons, GUT.

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1. Introduction

Models with fermions in bifundamental representations of product gauge groups of the general form $SU(a) \times SU(b) \times SU(c)$ have been studied for a variety of reasons over the last three decades [1]. The fermions must be free of gauge anomalies. The relevant gauge anomalies here are $SU(a)^3$, $SU(b)^3$ and $SU(c)^3$ (unless either a, b or c =2). So if the fermions are composed solely of bifundamental representations, they must be of the form

$$\frac{1}{d}[c(a, \bar{b}, 1) \oplus a(1, b, \bar{c}) \oplus b(\bar{a}, 1, c)] \tag{1.1}$$

or a multiple thereof. Here, d is the lowest common divisor of a, b and c . A simple example is the Trinification (TR) model [2] where $a = b = c = d = 3$ and a single anomaly family lives in the bifundamental representations $R_{\text{TR}} = (3, \bar{3}, 1) \oplus (1, 3, \bar{3}) \oplus (\bar{3}, 1, 3)$ of the gauge group $G_{\text{TR}} = SU(3) \times SU(3) \times SU(3)$. Another example is the Pati-Salam (PS) model where the gauge group is $G_{\text{PS}} = SU(4) \times SU(2) \times SU(2)$ [3] and the fermions representations are $R_{\text{PS}} = (4, 2, 1) \oplus (\bar{4}, 1, 2)$. This is an exception to the above general remarks since we do not need to include $(1, 2, \bar{2})$ because there are no $SU(2)^3$ gauge anomalies. This only happens when the gauge group contains $SU(2)$ subfactors. We still have $SU(2)$ global anomalies to worry about, but there are an even number of doublets, and so, the model is consistent.

In [4] we introduced a model based on the gauge group $SU(4) \times SU(3) \times SU(3)$ (the 334-model). It is the minimal model that contains both the (PS) model and the (TR) model. The 334-model contains features that are not present in many grand unified models. These include fractionally charged color singlet states and light magnetic monopoles. The richness of possible charge assignments were not explored in [4], but here we begin a more comprehensive analysis.

If string theory is to be the ultimate physical theory, then we must be able to extract standard model (SM) physics from it. A number of attempts with this objective have been pursued, including Calabi-Yau compactifications of the heterotic string, which yield E_6 type GUT theories, where holomorphic deformations, Wilson loops, etc., can be used to reduce the gauge symmetry. Orbifolding of type IIB strings on $AdS_5 \times S^5$ can produce four dimensional conformal field theories ($CFTs$) with gauge groups $\prod_i SU(Nd_i)$ and bifundamental matter [5, 6] and hence resemble the standard model. This is part of our motivation for studying the 334 model.

Here we again [4] take a bottom-up approach and consider models that are likely to be derivable from orbifolded type IIB strings, but our focus will be the resulting phenomenology, not on a string theory derivation of the model. The models we study contains aspects of both Calabi-Yau and AdS/CFT type string theory compactifications, and leads to a remarkably rich set of phenomenologies. It is well known that the (PS) model and the (TR) model are both contained in E_6 Grand Unification [7, 8]. We will provide additional coverings of PS and TR which do not embed in E_6 , but instead *require* nontrivial family unification and are perhaps the minimal such example of models with this property. We conclude this section with a brief review of PS and TR models. In the following sections, we present our models and then consider some of their phenomenological consequences.

The gauge group of the PS model is $G_{\text{PS}} = \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R$. Each fermion family lives in a set of bifundamental representation

$$(4, 2, 1) \oplus (\bar{4}, 1, 2). \tag{1.2}$$

This model embeds naturally in $\text{SO}(10)$ (as $G_{\text{PS}}/\mathbb{Z}_2$), where a fermion family combines into a **16** of $\text{SO}(10)$. Adding a $\mathbf{10} \oplus \mathbf{1}$ of fermions then allows unification into E_6 , where the fermions are now in a **27**. The PS, $\text{SO}(10)$, and E_6 models are all chiral and anomaly free, family by family, and so a full three-family [9] model is gotten simply by replicating the first family.

The TR model also has fermions in bifundamental representations

$$(3, \bar{3}, 1) \oplus (\bar{3}, 1, 3) \oplus (1, 3, \bar{3}) \tag{1.3}$$

where now the gauge group is $G_{\text{TR}} = \text{SU}(3) \times \text{SU}(3) \times \text{SU}(3)$. As $G_{\text{TR}}/\mathbb{Z}_3$ is a subgroup of E_6 and (3) already contains 27 fermionic states, the unification into E_6 is gotten simply by adding the necessary gauge generators to extend $\text{SU}(3)^3$ to E_6 . Again, a single fermion family is anomaly free on its own, so we must add two more families to agree with phenomenology.

2. Review of the $\text{SU}(4) \times \text{SU}(3) \times \text{SU}(3)$ model

The smallest group that contains both the PS and TR models is not E_6 but $G = \text{SU}(4) \times \text{SU}(3) \times \text{SU}(3)$, which has 31 generators and has a rank of 7. Insisting on fermions in bifundamental representations, we consider $(4, \bar{3}, 1)$, $(\bar{4}, 1, 3)$, and $(1, 3, \bar{3})$. We cannot take one of each to form a family, since this would be anomalous. The minimal anomaly free choice is [4].

$$3(4, \bar{3}, 1) \oplus 3(\bar{4}, 1, 3) \oplus 4(1, 3, \bar{3}). \tag{2.1}$$

If we break the $\text{SU}(4)$ to $\text{SU}(3)$, then (4) becomes

$$3[(3, \bar{3}, 1) \oplus (\bar{3}, 1, 3) \oplus (1, 3, \bar{3})] \oplus (1, \bar{3}, 1) \oplus (1, 1, 3) \oplus (1, 3, \bar{3}) \tag{2.2}$$

which contains three TR families plus a few additional particles. Hence the simplest set of anomaly free chiral bilinear representation [*i.e.*, (4)] contains three families. This is a true family unification, instead of a model where the second and third families are gotten from merely replicating the first. Examples of the latter include minimal $\text{SU}(5)$, $\text{SO}(10)$ with fermionic **16**s, and E_6 grand unification with fermions in **27**s, in addition to the PS and TR models.

The full analysis of the 334-model [10] requires the study of all the various patterns of spontaneous symmetry breaking (SSB), and the charge assignments these lead to, plus the phenomenological implication of the “extra” fermions. Typically there exist fractional charged color singlets [11, 12] in these models, and hence the minimal monopole charge will be the inverse of this minimal fraction times the Dirac charge.

As we shall see, there are only three inequivalent possibilities for embedding color and weak isospin of the standard model in $\text{SU}(4) \times \text{SU}(3) \times \text{SU}(3)$ (if we ignore the diagonal

subgroups). The embedding of weak hypercharge is more complicated. Consider the breaking $SU(4) \times SU(3)_A \times SU(3)_B \rightarrow SU(4) \times \frac{SU(2)_L \times U(1)_A}{\mathbb{Z}_2} \times \frac{SU(2)_R \times U(1)_B}{\mathbb{Z}_2}$. If we then break $U(1)_A$ and $U(1)_B$ completely, the hypercharge must be $Y = T_{3R} + (B - L)$, where T_{3R} is the diagonal generator of $SU(2)_R$, and $B - L$ generates the $U(1)$ that is in $SU(4)$ but not in $SU(3)_C$. However, there are many other possibilities for the embedding of $U(1)_Y$. These are similar to the well-known flipped models [13, 14]. One obvious choice is to break $SU(4)$ to $SU(3)_C$ and then Y could be the trinification choice from $SU(3)_L \times SU(3)_R$. Trinification has a standard hypercharge assignment, but this could be flipped. Also moving $SU(2)_W$ from $SU(4)$ to an $SU(3)$ of the 334-model corresponds to an isoflipped model [15]. (There are even more choices where family members move around and the charges of the extra fermions change. These “transflipped” models will be described in the next section.) Here we restrict ourselves to the standard hypercharge embeddings that generate PS and TR models, but keep in mind that flipping may offer other opportunities. (Note, embedding $SU(3)_C$ and/or $SU(2)_W$ in some diagonal subgroup within the 334-model, and this includes the “diagonal embedding” $SU(2)_W = SU(2)_{\text{diag}} \subset SU(2) \times SU(2) \subset SU(4)$, leads to vectorlike fermions, and this route is incompatible with SM phenomenology.)

We now begin our review of the most straightforward embedding followed in [4] which leads to the TR and PS models. In the next section we give a general analysis of all embeddings where the patterns of spontaneous symmetry breaking can be consistent with SM phenomenology.

The standard PS version of the 334-model, has fermions

$$3[(4, \bar{2}, 1) \oplus (4, \bar{1}, 1)] \oplus 3[(\bar{4}, 1, 2) \oplus (\bar{4}, 1, 1)] \oplus 4[(1, 2, \bar{2}) \oplus (1, 2, 1) \oplus (1, 1, \bar{2}) \oplus (1, 1, 1)] \quad (2.3)$$

Only the three PS families remain chiral, while the extra (exotic) vectorlike fermions obtain masses from Higgs VEVs at the G breaking scale. This will be discussed in more detail in the section on the Higgs sector. Breaking G_{PS} to the standard model gauge group $G_{\text{SM}} = \frac{SU(3)_C \times SU(2)_W \times U(1)_Y}{\mathbb{Z}_3}$, we find

$$\begin{aligned} &3[(3, 2)_{\frac{1}{6}} + (1, 2)_{-\frac{1}{2}} + (3, 1)_{\frac{1}{6}} + (1, 1)_{-\frac{1}{2}}] \\ &+ 3[(\bar{3}, 1)_{\frac{1}{3}} + (\bar{3}, 1)_{-\frac{2}{3}} + (1, 1)_1 + (1, 1)_0 + (\bar{3}, 1)_{-\frac{1}{6}} + (1, 1)_{\frac{1}{2}}] \\ &+ 4[(1, 2)_{-\frac{1}{2}} + (1, 2)_{\frac{1}{2}} + (1, 2)_0 + (1, 1)_{-\frac{1}{2}} + (1, 1)_{\frac{1}{2}} + (1, 1)_0.] \end{aligned}$$

As expected, we are left with three standard model families, plus three right-handed neutrinos, needed to form the three PS families, plus the following extra fermion states:

$$3[(3, 1)_{\frac{1}{6}} + (\bar{3}, 1)_{-\frac{1}{6}}] + 4[(1, 2)_{\frac{1}{2}} + (1, 2)_{-\frac{1}{2}}] + 7[(1, 1)_{\frac{1}{2}} + (1, 1)_{-\frac{1}{2}}] + 4(1, 2)_0 + 4(1, 1)_0.$$

Electric charge is now quantized in units of $\frac{1}{2}$, so the magnetic monopole must have a minimum charge of two from the Dirac quantization condition.

Care must be taken with the direct route to the TR model. This is due to a subtlety that arises in breaking $SU(4)$ to $SU(3)$. With the standard trinification charge assignments, one finds that massless charged quarks and leptons appear in the spectrum – a conflict with phenomenology. To avoid this we must include additional fermion multiplets at the 334

level. Let us see how this works. Two inequivalent cases must be considered: (i) $SU(3)_C$ embedded in $SU(4)$, or (ii) $SU(3)_C$ identified with an $SU(3)$ of the 334-model. In both cases at the trification level we begin with fermions as in (4).

For case (i), we have the three standard families plus

$$R_E = 3(1, \bar{3}, 1) \oplus 3(1, 1, 3) \oplus (1, 3, \bar{3}) \quad (2.4)$$

under $SU(3)_C \times SU(3)_L \times SU(3)_R$. Hence all the extra states are leptonic. Then for $SU(3)_L \times SU(3)_R \rightarrow \frac{SU(2)_L \times U(1)_L}{\mathbb{Z}_2} \times U(1)_R$ where we identify $U(1)_R$ with the diagonal generator $Y_R = \text{diag}(1, 1, -2)$ of $SU(3)_R$ and likewise $U(1)_L$ with the generator $Y_L = \text{diag}(1, 1, -2)$ of $SU(3)_L$, we can choose the hypercharge to be $Y = \frac{1}{6}Y_L + \frac{1}{3}Y_R$. The families just have the standard **27** of E_6 charges, while the new leptons are

$$5(1, 2)_{-\frac{1}{6}} + (1, 2)_{\frac{5}{6}} + 10(1, 1)_{\frac{1}{3}} + 5(1, 1)_{-\frac{2}{3}} \quad (2.5)$$

These states are still chiral, and the only way to give them mass would be with a VEV from an electrically charged Higgs. As this must obviously be avoided, an alternative, if we relax our restriction on only having bifundamental fermions, is to arrange these particles to be vectorlike by adding the conjugate, but anomaly free chiral multiplets

$$\bar{R}_E = 3(1, 3, 1) + 3(1, 1, \bar{3}) + (1, \bar{3}, 3) \quad (2.6)$$

at the 334 level. But this particular combination of R_E and \bar{R}_E contains the vectorlike pair of bifundamentals $(1, 3, \bar{3}) \oplus (1, \bar{3}, 3)$ which is also anomaly free since it is vectorlike. Barring any additional symmetry or some other mechanism, we would expect this pair to acquire a mass much higher than the 334 breaking scale, which will effectively lead to its decoupling. This is possible because there is a smaller anomaly free combination (since we now have fundamentals in addition to bifundamentals).

$$3(4, \bar{3}, 1) \oplus 3(\bar{4}, 1, 3) \oplus 3(1, 3, \bar{3}) \oplus 3(1, 3, 1) \oplus 3(1, 1, \bar{3}) \quad (2.7)$$

But this is none other than three copies of the anomaly free combination:

$$(4, \bar{3}, 1) \oplus (\bar{4}, 1, 3) \oplus (1, 3, \bar{3}) \oplus (1, 3, 1) \oplus (1, 1, \bar{3}) \quad (2.8)$$

Unfortunately, this means that the case where 334 breaks down via TR does not explain why there are three families.

Now, with fermions as in (2.7), upon breaking $G \rightarrow G_{\text{TR}}$ at some high scale $\langle \phi \rangle$, the chiral families stay massless while the extra fermions acquire mass terms of the form $h\langle \phi \rangle \bar{R}_E R_E$, where h is a typical Yukawa coupling constant. Hence, the fractionally charged leptons become heavy compared to the family fermions. Let us summarize the extra vectorlike leptons. There are five doublets with electric charges $\pm\frac{1}{3}$ and $\mp\frac{2}{3}$, two doublets with electric charge $\pm\frac{1}{3}$ and $\pm\frac{4}{3}$, ten singlets with $\pm\frac{1}{3}$ charges, and five singlets with $\mp\frac{2}{3}$ charges. The minimal monopole charge is three.

For case (ii), some of the extra states will be colored. In terms of $SU_C(3) \times SU_L(3) \times SU_R(3)$, they are

$$S_E = 3(3, 1, 1) + 3(1, 1, \bar{3}) + (\bar{3}, 1, 3). \quad (2.9)$$

The hypercharge is unchanged from case (1) (it is still $Y = \frac{1}{6}Y_L + \frac{1}{3}Y_R$), so now we find:

$$S_E = 3(3, 1)_0 + 3(1, 2)_{-\frac{1}{3}} + 3(1, 1)_{\frac{2}{3}} + (\bar{3}, 2)_{\frac{1}{3}} + (\bar{3}, 1)_{-\frac{2}{3}}. \quad (2.10)$$

Again we must add conjugate states

$$\bar{S}_E = 3(\bar{3}, 1, 1) + 3(1, 1, 3) + (3, 1, \bar{3}), \quad (2.11)$$

and this allows masses for the exotics at a scale higher than the family masses.

A point worth emphasizing is that unwanted Goldstone bosons do not appear when masses are given to the exotic fermions in the 334 model. First recall that Goldstone bosons arise when a generator of a symmetry of the lagrangian is broken. (There is one Goldstone for every generator that does not preserve the vacuum.) If these are gauge generators, the Goldstone is eaten via the higgs mechanism, otherwise it is a true Goldstone (massless scalar) that remains in the low energy spectrum. One further point worth recalling is that if there is a symmetry G^* of the Higgs potential V that is larger than the gauge group G (i.e., when G is a subgroup of G^*) then breaking a generator of G^* that is not in G results in a pseudo-Goldstone boson (PGB). Usually $G^* = G$, but G^* can be larger than G due to an accidental symmetry, or to an additional symmetry that can be imposed on V . A classic example is an $SU(3)$ gauge theory with a single adjoint ϕ . The generic Higgs potential is only $SU(3)$ invariant, but if one imposes a Z_2 symmetry $\phi \rightarrow -\phi$, the cubic term in V is eliminated and the Higgs potential becomes $SO(8)$ invariant. A VEV in the λ_8 component of ϕ then results in the gauge symmetry breaking to $SU(2) \times U(1)$ with four Goldstones being eaten. From the $SO(8)$ point of view, seven generators are broken, four correspond to those eaten by the Higgs mechanism, while the other three are massless at tree level, but these pseudo-Goldstones get masses at the one loop level. The remaining scalar state is heavy at tree level like a standard Higgs particle.

Any true Goldstones are always eaten in any spontaneously broken gauge theories where the symmetry of the full lagrangian is just the gauge symmetry G . If the Higgs potential has a higher symmetry (either imposed or accidental), then PGBs can arise, but they always get masses through loop corrections. PGBs are the exception rather than the rule, especially in models with complicated Higgs potentials like the ones we will be considering. We will always have multiple Higgs representations in our models, and so there will typically be cross terms, cubic terms, etc. that leave no room for symmetry beyond G . So to conclude, upon SSB, all Higgs fields are either eaten by the Higgs mechanism, or are heavy with mass on the order of the VEV . On the rare occasion when V has an accidental symmetry, the PGBs that arise would develop masses, typically at one loop.

With this review under our belt we are now ready to survey the complete list of allowed patterns of spontaneous symmetry breaking for the 334 model that lead to standard model physics plus extended families of fermions. As there are alternative hypercharge and weak isospin assignments (the flipped and isoflipped models) in $SO(10)$ and E_6 , so there are alternative embeddings of hypercharge and weak isospin in $SU(4) \times SU(3)_L \times SU(3)_R$ that are not the same as the PS or TR assignments discussed above. Recall that we will call these alternative hypercharge and weak isospin assignments “transflipped 334 models.” In

this case the flipping rearranges the location of the family members while keeping their charges fixed, but the extra fermions change both their location and charge. This is more general than the behavior of flipping in $SO(10)$ or E_6 where rearrangement takes place, but charges of extra fermions remain fixed.

3. Classification of inequivalent 334-models

There are three inequivalent ways to embed $SU(3)_C$ and $SU(2)_W$ in $SU(4) \times SU(3)_L \times SU(3)_R$ that can lead to the correct fermion families in the standard model group $SU(3)_C \times SU(2)_W \times U(1)_Y$.

- (i) Embed $SU(3)_C$ in $SU(4)$ and $SU(2)_W$ in either $SU(3)_L$ or $SU(3)_R$.
- (ii) Identify $SU(3)_C$ with $SU(3)_L$ and embed $SU(2)_W$ in $SU(3)_R$ (or vice versa).
- (iii) Identify $SU(3)_C$ with either $SU(3)_L$ or $SU(3)_R$ and let $SU(2)_W \subset SU(4)$.

In all cases the fermion representations at the 334 unification scale are $3(4, \bar{3}, 1) \oplus 3(\bar{4}, 1, 3) \oplus 4(1, 3, \bar{3})$, but in some circumstances it may be necessary to include additional fundamental or bifundamental representations to avoid unwanted massless charged particles.

In case (i) the initial $SU(4)$ contains a diagonal generator $\Lambda_{15} = \frac{1}{\sqrt{6}} \text{diag}(1, 1, 1, -3)$ orthogonal to $SU(3)_C$ [16].

With $SU(2)_W$ in $SU(3)_L$ we have another commuting generator $\lambda_8^L = \frac{1}{\sqrt{3}} \text{diag}(1, 1, -2)$, and there are two more diagonal $U(1)$ generators $\lambda_3^L = \text{diag}(1, -1, 0)$ and $\lambda_8^R = \frac{1}{\sqrt{3}} \text{diag}(1, 1, -2)$ in $SU(3)_R$. We now require the weak hypercharge $U(1)_Y$ be generated by a linear combination of Λ_{15} , λ_8^L , λ_3^R and λ_8^R such that we arrive at three families plus additional states. We can proceed systematically by following the decomposition of the fermion bifundamentals for

$$SU(4) \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_C \times U(1)_{\Lambda_{15}} \times SU(2)_W \times U(1)_{\lambda_8^L} \times U(1)_{\lambda_3^R} \times U(1)_{\lambda_8^R}$$

to find

$$(4, \bar{3}, 1) \rightarrow (3, \bar{2}, 1)_{1-100} + (1, \bar{2}, 1)_{-3-100} + (3, 1, 1)_{1200} + (1, 1, 1)_{-3200} \quad (3.1)$$

$$(1, 3, \bar{3}) \rightarrow (1, 2, 1)_{01-1-1} + (1, 2, 1)_{011-1} + (1, 2, 1)_{0102} \\ + (1, 1, 1)_{0-2-1-1} + (1, 1, 1)_{0-21-1} + (1, 1, 1)_{0-202} \quad (3.2)$$

$$(\bar{4}, 1, 3) \rightarrow (\bar{3}, 1, 1)_{-1011} + (\bar{3}, 1, 1)_{-10-11} + (\bar{3}, 1, 1)_{-100-2} \\ + (1, 1, 1)_{3011} + (1, 1, 1)_{30-11} + (1, 1, 1)_{300-2} \quad (3.3)$$

We now must solve for the generator of $U(1)_Y$. We do this by solving for the coefficients in

$$Y = a\Lambda_{15} + b\lambda_8^L + c\lambda_3^R + d\lambda_8^R \quad (3.4)$$

a	b	c	d	type
0	-1/6	-1/2	-1/6	TR
1/6	0	1/2	0	PS
-4/3	-3/2	-1/2	-3/2	I
-1/3	-1/2	1	0	II
5/12	1/4	-1/2	1/4	III
2/3	1/2	1	0	IV

Table 1: The six possible models of embedding class (i). Both the Pati-Salam model and the trinification model are of this class.

by identifying the families in the 334 bifundamentals. To start the process, note that in a standard family there is only one quark doublet, i.e., the $(3, 2)_{\frac{1}{6}}$. Therefore, comparing with the decomposition of the $(4, \bar{3}, 1)$ we must have

$$a - b = 1/6 \tag{3.5}$$

where we take a, b, c and d to be the coefficients of the unnormalized version of the U(1) generators Λ_{15} , λ_8^L , λ_3^R and λ_8^R .

There are three sets of $(\bar{3}, 1)$ s, so we can get two more conditions on the coefficients in the definition of Y by setting one of the three corresponding linear combinations (i.e., $-a + c + d$, $-a - c + d$, or $-a - 2d$) equal to the family hypercharge for the $(\bar{3}, 1)_{-2/3}$ and another to the hypercharge of the $(\bar{3}, 1)_{1/3}$. At this stage there is still one free parameter, but we need to check that sufficient freedom remains to have three $(1, 2)_{-1/2}$ s and three $(1, 1)_1$ s. By setting lepton doublet charges to $-\frac{1}{2}$ or singlet charges to 1 we find the allowed solutions summarized in table 1.

Other choices of coefficients correspond to trivial flippings. For instance, there are three equivalent choices for the overall diagonal generator of $SU(3)_R$ that enters Y , and they are given by (1.) $c = -1/2$ and $d = -1/6$, (2.) $c = 1/2$ and $d = -1/6$, and (3.) $d = 1/3$ with $c = 0$. (In E_6 this type of flipping is nontrivial if the spontaneous symmetry breaking is such that the states end up in different irreps of the decomposition, e.g., different routes through $SU(5)$ irreps.) As we mentioned above, the 334 model has nontrivial transflipping where the family states move about while the charges of the extra states change, and these are model I through IV in table I. As we already know the content of the TR and PS models we derived, let us summarize the fermion contents of the new models I through IV. They are

Case I. The coefficients are

$$(a, b, c, d) = (-4/3, -3/2, -1/2, -3/2). \tag{3.6}$$

They yield a weak hypercharge operator

$$Y_I = \underbrace{\begin{pmatrix} -\frac{4}{3} & 0 & 0 & 0 \\ 0 & -\frac{4}{3} & 0 & 0 \\ 0 & 0 & -\frac{4}{3} & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}}_{\text{SU}(4)} + \underbrace{\begin{pmatrix} -\frac{3}{2} & 0 & 0 \\ 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{\text{SU}(3)_L} + \underbrace{\begin{pmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{\text{SU}(3)_R}, \quad (3.7)$$

and fermions

$$(4, \bar{3}, 1) \rightarrow \underline{(3, 2)}_{1/6} + (1, 2)_{11/2} + (3, 1)_{-13/3} + \underline{(1, 1)}_1 \quad (3.8)$$

$$(1, 3, \bar{3}) \rightarrow (1, 2)_{1/2} + \underline{(1, 2)}_{-1/2} + (1, 2)_{-9/2} \\ + (1, 1)_5 + (1, 1)_4 + (1, 1)_0 \quad (3.9)$$

$$(\bar{4}, 1, 3) \rightarrow \underline{(\bar{3}, 1)}_{-2/3} + \underline{(\bar{3}, 1)}_{1/3} + (\bar{3}, 1)_{13/3} \\ + (1, 1)_{-6} + (1, 1)_{-5} + (1, 1)_{-1}, \quad (3.10)$$

where we have underlined the locations of members of the standard families.

Case II. Now the coefficients are

$$(a, b, c, d) = (-1/3, -1/2, 1, 0), \quad (3.11)$$

yielding

$$Y_{II} = \underbrace{\begin{pmatrix} -\frac{1}{3} & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{SU}(4)} + \underbrace{\begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{SU}(3)_L} + \underbrace{\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{SU}(3)_R}, \quad (3.12)$$

and fermions

$$(4, \bar{3}, 1) \rightarrow \underline{(3, 2)}_{1/6} + (1, 2)_{3/2} + (3, 1)_{-4/3} + (1, 1)_0 \quad (3.13)$$

$$(1, 3, \bar{3}) \rightarrow (1, 2)_{-3/2} + (1, 2)_{1/2} + \underline{(1, 2)}_{-1/2} \\ + (1, 1)_0 + (1, 1)_0 + \underline{(1, 1)}_1 \quad (3.14)$$

$$(\bar{4}, 1, 3) \rightarrow \underline{(\bar{3}, 1)}_{4/3} + \underline{(\bar{3}, 1)}_{-2/3} + \underline{(\bar{3}, 1)}_{1/3} \\ + (1, 1)_0 + (1, 1)_{-2} + (1, 1)_{-1}, \quad (3.15)$$

where we have again underlined the locations of members of the standard families. Note some of them have changed location from model I and the extra fermions have changed their locations as well as their charges.

Case III. The coefficients are

$$(a, b, c, d) = (5/12, 1/4, -1/2, 1/4), \quad (3.16)$$

leading to the hypercharge operator

$$Y_{\text{III}} = \underbrace{\begin{pmatrix} \frac{5}{12} & 0 & 0 & 0 \\ 0 & \frac{5}{12} & 0 & 0 \\ 0 & 0 & \frac{5}{12} & 0 \\ 0 & 0 & 0 & -\frac{5}{4} \end{pmatrix}}_{\text{SU}(4)} + \underbrace{\begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{\text{SU}(3)_L} + \underbrace{\begin{pmatrix} -\frac{1}{4} & 0 & 0 \\ 0 & \frac{3}{4} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{\text{SU}(3)_R}, \quad (3.17)$$

and fermions

$$(4, \bar{3}, 1) \rightarrow \underline{(3, 2)}_{1/6} + (1, 2)_{-3/2} + (3, 1)_{11/12} + (1, 1)_{-3/4} \quad (3.18)$$

$$(1, 3, \bar{3}) \rightarrow (1, 2)_{1/2} + \underline{(1, 2)}_{-1/2} + (1, 2)_{3/4} \\ + (1, 1)_{-1/4} + (1, 1)_{-5/4} + (1, 1)_0 \quad (3.19)$$

$$(\bar{4}, 1, 3) \rightarrow \underline{(\bar{3}, 1)}_{-2/3} + \underline{(\bar{3}, 1)}_{1/3} + \underline{(\bar{3}, 1)}_{-11/12} \\ + \underline{(1, 1)}_1 + (1, 1)_2 + (1, 1)_{3/4}, \quad (3.20)$$

$$(3.21)$$

with the locations of standard family members underlined. Remarkably, except for the positron, they are in the same places as they were in model II. The charges of the extra fermions have changes as expected.

Case IV. This time, the coefficients

$$(a, b, c, d) = (2/3, 1/2, 0, 1) \quad (3.22)$$

yield

$$Y_{\text{IV}} = \underbrace{\begin{pmatrix} \frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}}_{\text{SU}(4)} + \underbrace{\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{\text{SU}(3)_L} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{\text{SU}(3)_R}, \quad (3.23)$$

and fermions

$$(4, \bar{3}, 1) \rightarrow \underline{(3, 2)}_{1/6} + (1, 2)_{-1/2} + (3, 1)_{5/3} + (1, 1)_{-1}, \quad (3.24)$$

$$(1, 3, \bar{3}) \rightarrow \underline{(1, 2)}_{-1/2} + (1, 2)_{1/2} + (1, 2)_{1/2} \\ + (1, 1)_{-2} + (1, 1)_0 + (1, 1)_{-1}, \quad (3.25)$$

$$(\bar{4}, 1, 3) \rightarrow \underline{(\bar{3}, 1)}_{1/3} + \underline{(\bar{3}, 1)}_{5/3} + \underline{(\bar{3}, 1)}_{-2/3} \\ + (1, 1)_3 + \underline{(1, 1)}_1 + (1, 1)_2, \quad (3.26)$$

where the locations of members of the standard families have moved and the extra charges have changed values once again. In all cases the standard model family is distributed amongst all three types of initial 334 model bifundamental fermion representations.

As we have seen, embedding $\text{SU}(3)_C$ in $\text{SU}(4)$ (embedding class (i)) can lead to the PS model, but this is certainly not the case for embedding classes (ii) and (iii). However, the

\tilde{a}	\tilde{b}	\tilde{c}	\tilde{d}	type
0	1/3	0	-1/6	TR
1/2	$\frac{1-2x}{6}$	$\frac{x}{3}$	-1/6	X

Table 2: Class (ii) models.

TR model is allowed by all three embedding classes. We now move on the Class (ii) where $SU(3)_C$ is in an $SU(3)$ of the 334 model and $SU(2)_W$ is in the other $SU(3)$. The fermions decompose as

$$(4, \bar{3}, 1) \rightarrow (\bar{3}, 1)_{-1-1-10} + (\bar{3}, 1)_{1-1-10} + (\bar{3}, 1)_{02-10} + (\bar{3}, 1)_{0030}, \quad (3.27)$$

$$(1, 3, \bar{3}) \rightarrow (3, 2)_{000-1} + (3, 1)_{000-2}, \quad (3.28)$$

$$\begin{aligned} (\bar{4}, 1, 3) \rightarrow & (1, 2)_{1111} + (1, 1)_{111-2} \\ & + (1, 2)_{-1111} + (1, 1)_{-111-2} \\ & + (1, 2)_{0-211} + (1, 1)_{0-21-2} \\ & + (1, 2)_{00-31} + (1, 1)_{00-3-2}, \end{aligned} \quad (3.29)$$

and the generator of $U(1)_Y$ is

$$Y = \tilde{a}\Lambda_3 + \tilde{b}\Lambda_8^L + \tilde{c}\Lambda_{15}^R + \tilde{d}\lambda_8^R. \quad (3.30)$$

We start the process of identifying the families in the 334 bifundamentals by again noting there is only one type of quark doublet. Therefore, comparing with the decomposition of the $(4, \bar{3}, 1)$ we must have $\tilde{d} = -1/6$. Again requiring the existence of $(\bar{3}, 1)$ s with hypercharges 1/3 and $-2/3$ and leptons $(1, 2)_{-1/2}$ and $(1, 1)_1$ leads to a set of equations for the coefficients in (43) with solutions summarized in table 2.

As expected the TR model is a solution, however, the ‘‘X’’ model is a surprise. We find we can satisfy all the conditions necessary to fix the charge of all standard model family members without the need to specify the value of the parameter x in the hypercharge,

$$Y = \underbrace{\begin{pmatrix} \frac{2}{3} & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & x - \frac{1}{3} & 0 \\ 0 & 0 & 0 & -x \end{pmatrix}}_{SU(4)} + \underbrace{\begin{pmatrix} -\frac{1}{6} & 0 & 0 \\ 0 & -\frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}}_{SU(3)_R}. \quad (3.31)$$

I.e., the generator $X = \text{diag}(0, 0, 1, -1)$ of $U(1)_X \subset SU(4)$ is still at our disposal. Before

discussing this model, let us display the fermions:

$$(4, \bar{3}, 1) \rightarrow \underline{(\bar{3}, 1)_{-2/3}} + \underline{(\bar{3}, 1)_{1/3}} + (\bar{3}, 1)_{1/3-x} + (\bar{3}, 1)_x, \quad (3.32)$$

$$(1, 3, \bar{3}) \rightarrow \underline{(3, 2)_{1/6}} + (3, 1)_{-1/3}, \quad (3.33)$$

$$\begin{aligned} (\bar{4}, 1, 3) \rightarrow & \underline{(1, 2)_{1/2}} + \underline{(1, 1)_1} \\ & + \underline{(1, 2)_{-1/2}} + \underline{(1, 1)_0} \\ & + (1, 2)_{x-1/2} + (1, 1)_x \\ & + (1, 2)_{-x-1/6} + (1, 1)_{-x+1/3}. \end{aligned} \quad (3.34)$$

We find the three complete standard model families (underlined states) without specifying x . There are several choices for x that can be used to generate a flipped model. $x = 1, 0, -2/3$ or $1/3$ flip the $(\bar{3}, 1)$ s, $x = 0$ flips the $(1, 2)$ s, and $x = 3/2, 1, 4, 3$ or $-7/6$ flips the $(1, 1)$ s. The model does not, a priori, require complete charge quantization of the standard families relative to the extra fermions. In any model where $U(1)_X$ is spontaneously broken by vacuum expectation values for Higgs scalars in a representation of $SU(4)$, electric charge will end up quantized depending on the charges of that representation. However, it is not necessary for $U(1)_X$ to be spontaneously broken since all of the standard model fields are neutral under it, turning $U(1)_X$ into a hidden sector gauge symmetry.

To see this, define the generator Z by

$$Z = \underbrace{\begin{pmatrix} \frac{2}{3} & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{SU(4)} + \underbrace{\begin{pmatrix} -\frac{1}{6} & 0 & 0 \\ 0 & -\frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}}_{SU(3)_R} \quad (3.35)$$

Then, $\frac{Y}{2} = Z + xX$, which means that the subgroup generated by Y is a subgroup of $U(1)_Z \times U(1)_X$ and that subgroup cannot possibly be $U(1)_X$. If x is a rational number, then that subgroup is isomorphic to $U(1)$ and we can call it $U(1)_Y$ and the hypercharges, and hence the electric charges are quantized. On the other hand, if x is irrational, then that subgroup is dense in $U(1)_Z \times U(1)_X$. Since the model that we are dealing with is continuous under symmetry transformations, if we have an infinite sequence of gauge transformations leaving the vacuum [17] invariant that converges to some gauge transformation, then that gauge transformation also leaves the vacuum invariant. So, if Y is unbroken up to the electroweak breaking scale, then so is the closure of the subgroup generated by it; $U(1)_Z \times U(1)_X$ is also unbroken right up to the electroweak scale. The electric charge operator is $Q = \frac{Y}{2} + \frac{I}{2}$, where I is the weak isospin operator. Let us define $V = Z + \frac{I}{2}$, such that $Q = V + xX$. Since the electromagnetic symmetry is unbroken, by the same reasoning, this also means that $U(1)_V \times U(1)_X$ is also unbroken. Nothing in this discussion depends upon the details of the symmetry breaking (Higgs or dynamical).

However, there will be cross-coupling terms for the gauge kinetic terms. Let A_Z be the gauge field for $U(1)_Z$ and A_X be the gauge field for $U(1)_X$, with F_Z and F_X their

respective field strengths. The gauge kinetic term will have the generic form

$$-\frac{1}{4g_Z^2}F_Z^{\mu\nu}F_{Z\mu\nu} - \frac{1}{4g_Y^2}F_X^{\mu\nu}F_{X\mu\nu} - \frac{\alpha}{2}F_Z^{\mu\nu}F_{X\mu\nu} \tag{3.36}$$

where α is some dimensionless coefficient that receives contributions from radiative corrections.

By taking linear combinations X and $Y' = Z + \beta X$, we can diagonalize the gauge kinetic terms with a proper choice of β , but in that case, the Y' charges of SM fields and the exotic fields will be incommensurable. Since all the SM fields are neutral under X , A_X decouples from the low energy physics and so, it is not a problem that it remains unbroken. However, it is essential that all exotics with a nonzero X charge have large mass, since they couple to both the standard model gauge fields and A_X . This probably means that our current model does not work phenomenologically because, so far we have not been able to find a way to make all the exotic fermions vectorlike. However, this mechanism works generically for any GUT theory which contains $U(1)_Z \times U(1)_X$ with all the standard model fields being X -neutral and the Z charges of all the standard model fields coinciding with their hypercharges.

We are now ready to continue on to class (iii) models where $SU(2)$ is embedded in $SU(4)$. We can write the hypercharge operator as

$$Y = \hat{a}\Lambda_{8'} + \hat{b}\Lambda_{15'} + \hat{c}\lambda_3^R + \hat{d}\lambda_8^R, \tag{3.37}$$

where we define $\Lambda_{8'} = \text{diag}(1, 1, -1, -1)$ and $\Lambda_{15'} = \text{diag}(0, 0, 1, -1)$. The fermions now decompose as (writing the representations in the order $(SU(3)_C, SU(2)_W)$ to agree with the previous notation)

$$(\bar{4}, 1, 3) \rightarrow (3, 2)_{-1000} + (3, 1)_{1-100} + (3, 1)_{1100}. \tag{3.38}$$

$$(1, 3, \bar{3}) \rightarrow (\bar{3}, 1)_{0011} + (\bar{3}, 1)_{00-11} + (\bar{3}, 1)_{000-2}. \tag{3.39}$$

$$\begin{aligned} (4, \bar{3}, 1) \rightarrow & (1, 2)_{10-1-1} + (1, 2)_{101-1} + (1, 2)_{1002} \\ & + (1, 1)_{-11-1-1} + (1, 1)_{-111-1} + (1, 1)_{-110-2} \\ & + (1, 1)_{-1-1-1-1} + (1, 1)_{-1-11-1} + (1, 1)_{-1-102}. \end{aligned} \tag{3.40}$$

The usual process of requiring the quark doublets to have hypercharge $\frac{1}{6}$, the $(\bar{3}, 1)$ s have hypercharges $1/3$ and $-2/3$ and leptons $(1, 2)_{-1/2}$ and $(1, 1)_1$ leads to two inequivalent models where the hypercharge operator is either

$$Y_{\text{TR}} = \underbrace{\begin{pmatrix} -\frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{SU(4)} + \underbrace{\begin{pmatrix} -\frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}}_{SU(3)_R} \tag{3.41}$$

\hat{a}	\hat{b}	\hat{c}	\hat{d}	type
-1/6	1/6	1/2	-1/6	TR
-1/6	7/6	0	1/3	V

Table 3: Class (iii) models.

or

$$Y_V = \underbrace{\begin{pmatrix} -\frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{4}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}_{\text{SU}(4)} + \underbrace{\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}}_{\text{SU}(3)_R}. \quad (3.42)$$

Tabulating the coefficients we have table 3.

As the first case is the TR model again we need only consider the second. Here the fermions are

$$(\bar{4}, 1, 3) \rightarrow \underline{(3, 2)}_{1/6} + (3, 1)_{-4/3} + (3, 1)_1. \quad (3.43)$$

$$(1, 3, \bar{3}) \rightarrow \underline{(\bar{3}, 1)}_{1/3} + \underline{(\bar{3}, 1)}_{1/3} + \underline{(\bar{3}, 1)}_{-2/3}. \quad (3.44)$$

$$(4, \bar{3}, 1) \rightarrow \underline{(1, 2)}_{-1/2} + (1, 2)_{-1/2} + (1, 2)_{1/2} \\ + (1, 1)_{-2/3} + (1, 1)_{-2/3} + \underline{(1, 1)}_1 \\ + (1, 1)_{-1/3} + (1, 1)_{-1/3} + (1, 1)_{2/3}. \quad (3.45)$$

Multiple choices for selecting family components exist here of which we have underlined one possibility.

4. The Higgs sector

In this section, we will present a detailed analysis of the Higgs sector for the cases where the intermediate gauge group is the 422 (PS) group, the flipped 422 group and the trification group.

5. Pati-Salam models

Consider the symmetry breaking chain

$$\text{SU}(4) \times \text{SU}(3)_A \times \text{SU}(3)_B \rightarrow \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R \\ \rightarrow \frac{\text{SU}(3)_C \times \text{U}(1)_Y}{\mathbb{Z}_3} \times \text{SU}(2)_W,$$

which takes us from the 334 model to the standard model via the PS model, where we take

$$\text{SU}(2)_W = \text{SU}(2)_L \subset \text{SU}(3)_A, \\ \text{SU}(2)_R \subset \text{SU}(3)_B,$$

and

$$\mathrm{SU}(3)_C \subset \mathrm{SU}(4).$$

The weak hypercharge in this model is

$$Y_{\mathrm{PS}} = \underbrace{\begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{\mathrm{SU}(4)} + \underbrace{\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}}_{\mathrm{SU}(2)_R} = \underbrace{\begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{\mathrm{SU}(4)} + \underbrace{\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{\mathrm{SU}(3)_B},$$

and is of PS type.

The fact that the SM gauge group is modded out by \mathbb{Z}_3 instead of the usual \mathbb{Z}_6 [18] means that we can have electric charges which are multiples of $\frac{1}{2}$. At distances smaller than the QCD confinement scale, we can also have electric charges which are multiples of $\frac{1}{6}$, but at larger distances, we only find electric charges coming in multiples of $\frac{1}{2}$. However, these fractional charges are attached to particles with GUT scale¹ masses. Monopoles form at the PS breaking scale.

5.1 Model PS_α

5.1.1 Fermion content

The fermion content at the 334 level is $3[(4, \bar{3}, 1)_{433} \oplus (\bar{4}, 1, 3)_{433} \oplus (1, 3, \bar{3})_{433} \oplus (1, 3, 1)_{433} \oplus (1, 1, \bar{3})_{433}]$ [19] with decomposition of the representations under the PS group given by

$$\begin{aligned} (4, \bar{3}, 1)_{433} &\rightarrow (4, 2, 1)_{\mathrm{PS}} \oplus (4, 1, 1)_{\mathrm{PS}}, \\ (\bar{4}, 1, 3)_{433} &\rightarrow (\bar{4}, 1, 2)_{\mathrm{PS}} \oplus (\bar{4}, 1, 1)_{\mathrm{PS}}, \\ (1, 3, \bar{3})_{433} &\rightarrow (1, 2, 2)_{\mathrm{PS}} \oplus (1, 2, 1)_{\mathrm{PS}} \oplus (1, 1, 2)_{\mathrm{PS}} \oplus (1, 1, 1)_{\mathrm{PS}}, \\ (1, 3, 1)_{433} &\rightarrow (1, 2, 1)_{\mathrm{PS}} \oplus (1, 1, 1)_{\mathrm{PS}}, \\ (1, 1, \bar{3})_{433} &\rightarrow (1, 1, 2)_{\mathrm{PS}} \oplus (1, 1, 1)_{\mathrm{PS}}, \end{aligned}$$

We find the SM fermions (plus a left handed antineutrino) are contained within $(4, 2, 1)_{\mathrm{PS}}$ and $(\bar{4}, 1, 2)_{\mathrm{PS}}$. All the other fermions are “exotic” and must be heavy enough to have escaped detection.

5.1.2 Yukawa couplings

Our goal is to make the exotic fermions vectorlike. In general, this can be done either at the 433 breaking scale or at the PS breaking scale. Since all the PS representations of the fermion fields other than the Standard Model + left handed antineutrino either come in conjugate pairs or are real representations, it is much more economical to arrange for all the pairings to occur at the 433 breaking scale. Here, we assume that the pairings result

¹The GUT scale is defined to be the energy scale at which the GUT gauge symmetry is spontaneously broken.

from Yukawa terms after the Higgs field(s) acquire VEVs. Pairings are also possible using nonrenormalizable couplings [20] or other means like dynamical symmetry breaking.

Pairing $(4, 1, 1)_{\text{PS}}$ with $(\bar{4}, 1, 1)_{\text{PS}}$ requires either $\langle (15, 3, \bar{3})_{H433} \rangle$ [21] or $\langle (1, 3, \bar{3})_{H433} \rangle$. The choice of $15_{\text{SU}(4)}$ and $1_{\text{SU}(4)}$ comes about because they are the only two $\text{SU}(4)$ -representations ρ for which there exists a nonzero intertwiner² from $\rho \otimes 4_{\text{SU}(4)} \otimes \bar{4}_{\text{SU}(4)}$ to the singlet representation where the interaction terms are gauge invariant. The VEV lies along the $(1, 1, 1)_{\text{HPS}}$ component of the Higgs field. The second choice is simpler, but the first choice works just as well, at least when it comes to Yukawa couplings. However, as we will see later, a $(1, 3, \bar{3})_{H433}$ Higgs is necessary for other reasons. So, the first choice may be an unnecessary complication. In summary, we choose either

$$\langle (1, 3, \bar{3})_{H433} \rangle (4, \bar{3}, 1)_{433} (\bar{4}, 1, 3)_{433},$$

or

$$\langle (15, 3, \bar{3})_{H433} \rangle (4, \bar{3}, 1)_{433} (\bar{4}, 1, 3)_{433}.$$

$(1, 2, 2)_{\text{PS}}$ is real and the only $(1, 2, 2)_{\text{PS}}$'s come from $(1, 3, \bar{3})_{433}$. So, the pairing has to be Majorana. Hence,

$$\langle (1, 3, \bar{3})_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, \bar{3})_{433}$$

is the only choice [22].

Now let's move on to the $(1, 2, 1)_{\text{PS}}$ fermions. Neither

$$\langle (1, 3, 6)_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, \bar{3})_{433}$$

nor

$$\langle (1, 3, 1)_{H433} \rangle (1, 3, 1)_{433} (1, 3, 1)_{433}$$

are satisfactory because they lead to antisymmetric mass self-couplings of the fermion fields and with three generations, this still leaves us with some leftover massless fermions. Even if we have more than one such Higgs field with different Yukawa coupling constants between the generations, we are still left with massless fermions since the sum of two antisymmetric matrices is still an antisymmetric matrix which has a zero eigenvalue if its dimension is odd. It is not that we cannot add such couplings but that with such couplings, we still need to add the other Yukawa couplings to pair up all the exotics anyway and so, the addition of such couplings is an unnecessary complication, unless they are required to provide useful mass relations. This leaves us with the choices that avoid asymmetric mass matrices, i.e., we require

$$\langle (1, 3, 3)_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, 1)_{433}$$

to give masses to the $(1, 2, 1)_{\text{PS}}$ fermions. Similarly, for the $(1, 1, 2)_{\text{PS}}$ fermions, we need the coupling

$$\langle (1, \bar{3}, \bar{3})_{H433} \rangle (1, 3, \bar{3})_{433} (1, 1, \bar{3})_{433}.$$

Finally, we are left to deal with the $(1, 1, 1)_{\text{PS}}$ fermions. They leave us with the greatest degree of freedom. However, since they do not couple to the PS gauge fields, (and as a

²An intertwiner is a 433-invariant linear map from a representation into another.

consequence the SM gauge fields), and as all the SM fermions are located within $(4, \bar{3}, 1)_{433}$ and $(\bar{4}, 1, 3)_{433}$ and as these singlets are located within the SU(4)-neutral representations, we have not added a single Yukawa coupling which mixes the SU(4)-charged fermions with the SU(4)-neutral fermions, it is unnecessary to pair them up.

Note that all of the Higgs fields introduced previously with the possible exception of the optional $(15, 3, \bar{3})_{H433}$ are SU(4)-singlets. This means that we need additional Higgs fields which are not SU(4)-singlets to break PS down to SM. These Higgs fields will acquire VEVs at a lower energy scale compared to the SU(4)-neutral Higgs fields.

Since the electroweak doublets couple some SM components of $(4, \bar{3}, 1)_{433}$ to some SM components of $(\bar{4}, 1, 3)_{433}$, they have to lie in either $(15, 3, \bar{3})_{H433}$ or $(1, 3, \bar{3})_{H433}$ or some linear combination. As $(15, 3, \bar{3})_{H433}$ is not really needed, the second choice is the simplest one. (We still need to break SU(4) to $SU(3) \times U(1)$, and the most efficient choice for this is $(15, 1, 1)_{H433}$.) If only one $(1, 3, \bar{3})_{H433}$ field exists, this will lead to mass relations between the up-type and down-type quarks which are not observed. So, there has to be at least two such fields.

In a full phenomenological model, we will have to make sure that all the Higgs fields pair up except for the electroweak doublets (unless there is some mechanism making the additional Higgs fields phenomenologically harmless) with the possible exception of some SM-singlet Higgs fields (or even electroweak triplets, but then we might have to worry about protecting the tiny mass of the left-handed neutrino), but we will not work this out here. Higgs fields with SU(3)_C-color are likely to cause rapid proton decay.

Lastly, the left-handed antineutrino has to acquire a Majorana mass. There are number of ways to do this. For instance,

$$\langle (10, 1, \bar{6})_{H433} \rangle (\bar{4}, 1, 3)_{433} (\bar{4}, 1, 3)_{433}$$

is one of them. A Yukawa coupling with a Higgs VEV associated with the PS breaking scale of the left-handed antineutrino to one of the vector exotics which already has a mass associated with the 433 breaking scale is another. Or we can even add singlet fermion fields $(1, 1, 1)_{433}$ and pair up the left-handed antineutrino with it. Any of these methods will lead to a small seesaw mass for the left-handed neutrino.

5.2 Model PS β

5.2.1 Fermion content

Now consider the 334 model with pure bifundamental fermions, $3(4, \bar{3}, 1)_{433} \oplus 3(\bar{4}, 1, 3)_{433} \oplus 4(1, 3, \bar{3})_{433}$ and therefore family unification. These fermions decompose via

$$\begin{aligned} (4, \bar{3}, 1)_{433} &\rightarrow (4, 2, 1)_{PS} \oplus (4, 1, 1)_{PS}, \\ (\bar{4}, 1, 3)_{433} &\rightarrow (\bar{4}, 1, 2)_{PS} \oplus (\bar{4}, 1, 1)_{PS}, \end{aligned}$$

and

$$(1, 3, \bar{3})_{433} \rightarrow (1, 2, 2)_{PS} \oplus (1, 2, 1)_{PS} \oplus (1, 1, 2)_{PS} \oplus (1, 1, 1)_{PS}.$$

5.2.2 Yukawa couplings

The symmetry breaking can be analysed by using the same reasoning as in the previous model, where we choose either

$$\langle (1, 3, \bar{3})_{H433} \rangle (4, \bar{3}, 1)_{433} (\bar{4}, 1, 3)_{433},$$

or

$$\langle (15, 3, \bar{3})_{H433} \rangle (4, \bar{3}, 1)_{433} (\bar{4}, 1, 3)_{433},$$

to give masses to the $(4, 1, 1)_{\text{PS}}$ and $(\bar{4}, 1, 1)_{\text{PS}}$. The rest of the Yukawa terms are determined (after making the same assumptions as previously discussed) to be

$$\begin{aligned} &\langle (1, 3, \bar{3})_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, \bar{3})_{433}, \\ &\langle (1, 3, 6)_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, \bar{3})_{433}, \\ &\langle (1, \bar{6}, \bar{3})_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, \bar{3})_{433}. \end{aligned}$$

The fact that two of the couplings lead to antisymmetric mass matrices is no problem here since there are *four* $(1, 3, \bar{3})$ s and as before, we do not need to pair up $(1, 1, 1)_{\text{PS}}$.

The comment that additional Higgs fields are needed to break the PS gauge group also applies here. So do the comments about the electroweak Higgs doublets and giving the left-handed antineutrino a large Majorana mass. For instance, if we avoid the $\langle (15, 3, \bar{3})_{H433} \rangle$ above, then we need to include a $\langle (15, 1, 1)_{H433} \rangle$ to break $\text{SU}(4)$.

6. Model FPSa

The symmetry breaking chain is [23]

$$\begin{aligned} \text{SU}(4) \times \text{SU}(3)_A \times \text{SU}(3)_B &\rightarrow \text{SU}(4) \times \text{SU}(2)_L \times \frac{\text{SU}(2)_R \times \text{U}(1)_X}{\mathbb{Z}_2} \\ &\rightarrow \frac{\text{SU}(3)_C \times \text{U}(1)_Y}{\mathbb{Z}_3} \times \text{SU}(2)_W, \end{aligned}$$

where

$$\begin{aligned} \text{SU}(2)_W &= \text{SU}(2)_L \subset \text{SU}(3)_A, \\ \text{SU}(2)_R &\subset \text{SU}(3)_B, \end{aligned}$$

and

$$\text{SU}(3)_C \subset \text{SU}(4).$$

Defining the generator

$$Z = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}}_{\text{SU}(3)_B}$$

we can write the hypercharge in the form

$$Y = \underbrace{\begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{\text{SU}(4)} + \underbrace{\begin{pmatrix} \frac{1}{4} & 0 \\ 0 & -\frac{1}{4} \end{pmatrix}}_{\text{SU}(2)_R} + \frac{Z}{4} = \underbrace{\begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{\text{SU}(4)} + \underbrace{\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{\text{SU}(3)_B}$$

The embedding of the SM gauge group within 433 is the same as in the unflipped PS model (which means that they are the same type of model in our classification). The matter sector, the Higgs sector, and the Yukawa couplings are also the same. The difference lies in the relative values of the different Higgs VEVs, which determines the intermediate gauge group. This is not unlike the case of some SO(10) models, for example, where despite having the same matter and Higgs sectors and Yukawa couplings, the intermediate gauge group may be 622 or SU(5), depending upon the relative scale of $\langle 54_{H \text{SO}(10)} \rangle$ and $\langle 16_{H \text{SO}(10)} \rangle / \langle \overline{16}_{H \text{SO}(10)} \rangle$.

At distances smaller than the QCD confinement scale, we can again have electric charges which are multiples of $\frac{1}{6}$, but at larger distances, we only find electric charges in multiples of $\frac{1}{2}$. However, these fractional charges are for particles with GUT scale masses. Again, monopoles form at the 433 breaking scale.

6.1 Model FPS α

Again we must consider fermions with family replication

$$3[(4, \bar{3}, 1)_{433} \oplus (\bar{4}, 1, 3)_{433} \oplus (1, 3, \bar{3})_{433} \oplus (1, 3, 1)_{433} \oplus (1, 1, \bar{3})_{433}],$$

with decompositions

$$\begin{aligned} (4, \bar{3}, 1)_{433} &\rightarrow (4, 2, 1)_{0FPSA} \oplus (4, 1, 1)_{0FPSA}, \\ (\bar{4}, 1, 3)_{433} &\rightarrow (\bar{4}, 1, 2)_{1FPSA} \oplus (\bar{4}, 1, 1)_{-2FPSA}, \\ (1, 3, \bar{3})_{433} &\rightarrow (1, 2, 2)_{-1FPSA} \oplus (1, 2, 1)_{2FPSA} \oplus (1, 1, 2)_{-1FPSA} \oplus (1, 1, 1)_{2FPSA}, \\ (1, 3, 1)_{433} &\rightarrow (1, 2, 1)_{0FPSA} \oplus (1, 1, 1)_{0FPSA}, \end{aligned}$$

and

$$(1, 1, \bar{3})_{433} \rightarrow (1, 1, 2)_{-1FPSA} \oplus (1, 1, 1)_{2FPSA},$$

where we use the following array for definitions

$(4, 2, 1)_{0FPSA}$	$(\bar{4}, 1, 2)_{1FPSA}$	$(4, 1, 1)_{0FPSA}$	$(\bar{4}, 1, 1)_{-2FPSA}$
$\begin{pmatrix} (3, 2)_{\frac{1}{6}} & q \\ (1, 2)_{-\frac{1}{2}} & l \end{pmatrix}$	$\begin{pmatrix} (\bar{3}, 1)_{\frac{1}{3}} & d^c \\ (\bar{3}, 1)_{-\frac{1}{6}} & e^c \\ (1, 1)_1 & \\ (1, 1)_{\frac{1}{2}} & \end{pmatrix}$	$\begin{pmatrix} (3, 1)_{\frac{1}{6}} \\ (1, 1)_{-\frac{1}{2}} \end{pmatrix}$	$\begin{pmatrix} (\bar{3}, 1)_{-\frac{2}{3}} & u^c \\ (1, 1)_0 & \nu^c \end{pmatrix}$

The SM fermions are flipped, which is why this model is called a flipped PS model. Both $(4, 1, 2)_{1FPSA}$ and $(\bar{4}, 1, 1)_{0FPSA}$ contain additional exotics.

6.1.1 Yukawa couplings

Now either

$$\langle (1, 3, \bar{3})_{H433} \rangle (4, \bar{3}, 1)_{433} (\bar{4}, 1, 3)_{433}$$

or

$$\langle (15, 3, \bar{3})_{H433} \rangle (4, \bar{3}, 1)_{433} (\bar{4}, 1, 3)_{433}$$

is necessary to begin to carry out the SSB in this model. The former Higgs fields decompose as

$$(1, 3, \bar{3})_{H433} \rightarrow (1, 2, 2)_{-1HFPSA} \oplus (1, 2, 1)_{2HFPSA} \oplus (1, 1, 2)_{-1HFPSA} \oplus (1, 1, 1)_{2HFPSA},$$

which makes it clear that its VEVs are at the FPSA breaking scale, since none of its components are FPSA-singlets.

The Yukawa coupling responsible for pairing of exotics is

$$\langle (1, 1, 2)_{-1HFPSA} \rangle (4, 1, 1)_{0FPSA} (\bar{4}, 1, 2)_{1FPSA}$$

None of the other Yukawa terms lead to further pairings until the electroweak breaking scale. The down-type and up-type electroweak Higgs doublets are contained within $(1, 2, 2)_{-1HFPSA}$ and $(1, 2, 1)_{2HFPSA}$ respectively. The term $(1, 2, 2)_{-1HFPSA} (4, 2, 1)_{0FPSA} (\bar{4}, 1, 2)_{1FPSA}$ contains $(1, 2)_{-\frac{1}{2}H} qd^c$ and $(1, 2)_{-\frac{1}{2}H} le^c$, while $(1, 2, 1)_{2HFPSA} (4, 2, 1)_{0FPSA} (\bar{4}, 1, 1)_{-2FPSA}$ contains $(1, 2)_{\frac{1}{2}H} qu^c$ and $(1, 2)_{\frac{1}{2}H} l\nu^c$.

The decompositions of the other fermions are displayed below:

$(1, 3, \bar{3})_{433}$	$(1, 3, 1)_{433}$	$(1, 1, \bar{3})_{433}$
$\begin{pmatrix} (1, 2)_{-\frac{1}{2}} & (1, 1)_{-\frac{1}{2}} \\ (1, 2)_0 & (1, 1)_0 \\ (1, 2)_{\frac{1}{2}} & (1, 1)_{\frac{1}{2}} \end{pmatrix}$	$\begin{pmatrix} (1, 2)_0 & (1, 1)_0 \end{pmatrix}$	$\begin{pmatrix} (1, 1)_{-\frac{1}{2}} \\ (1, 1)_0 \\ (1, 1)_{\frac{1}{2}} \end{pmatrix}$

The $(1, 2)_{\frac{1}{2}}$ of $(1, 3, \bar{3})_{433}$ has to pair up with the $(1, 2)_{-\frac{1}{2}}$ of $(1, 3, \bar{3})_{433}$. This is done using the coupling

$$\langle (1, 3, \bar{3})_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, \bar{3})_{433}.$$

Since there are three generations of $(1, 3, \bar{3})_{433}$ and the self-pairing of $(1, 2)_0$ is anti-symmetric since it is a pseudoreal representation, we have to pair the $(1, 2)_0$ of $(1, 3, \bar{3})_{433}$ with the $(1, 2)_0$ of $(1, 3, 1)_{433}$. So, the coupling

$$\langle (1, 3, 3)_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, 1)_{433}$$

is required.

To pair up $(1, 1)_{\frac{1}{2}}$ and $(1, 1)_{-\frac{1}{2}}$, the following two couplings are unnecessary

$$\langle (1, \bar{6}, \bar{3})_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, \bar{3})_{433}$$

$$\langle (1, 1, \bar{3})_{H433} \rangle (1, 1, \bar{3})_{433} (1, 1, \bar{3})_{433}$$

for the same reason of antisymmetry and an odd number of generations.

Finally, we are left with the choice

$$\langle (1, \bar{3}, \bar{3})_{H433} \rangle (1, 3, \bar{3})_{433} (1, 1, \bar{3})_{433}.$$

There is no reason to pair up the SM-singlet fermions here either, so we are left with the same four Yukawa coupling terms as in the previous model.

Since SU(4) must be broken, there have to be Higgs fields which transform under SU(4) nontrivially and acquire VEVs at the FPSA breaking scale. Unless there is some principle (symmetry or otherwise) [24] or some dynamical mechanism preventing us, we might expect to have Yukawa (or nonrenormalizable) couplings involving this Higgs field, a SU(4)-charged fermion field and an SU(4)-neutral fermion field. An example is

$$\langle (\bar{4}, 1, 3)_{H433} \rangle (4, \bar{3}, 1)_{433} (1, 3, \bar{3})_{433}.$$

This can cause some further flipping so that the standard model fermions are really linear combinations of some components of $(4, \bar{3}, 1)_{433}$ and some components of $(1, 3, \bar{3})_{433}$. This modifies the electroweak Yukawa couplings so that we might get away with only one $(1, 3, \bar{3})_{H433}$ Higgs field, instead of two, and still get the right mass relations among the fermions. But in that case, we definitely need to pair up all the SM-singlet fermions or otherwise, there will be observable mixings. This is unlike the case of the unflipped PS model, because the leptons in the $(1, 3, \bar{3})_{433}$ (and also $(1, 3, 1)_{433}$ and $(1, 1, \bar{3})_{433}$ in some models) acquire 433 breaking scale masses and a PS scale cross-coupling between $(4, \bar{3}, 1)_{433}$ or $(\bar{4}, 1, 3)_{433}$ with $(1, 3, \bar{3})_{433}$ will only change the SM lepton eigenstates by a tiny angle.

6.2 Model FPSa β

This construction is similar to the previous model, so we shall not go through the details here.

7. Model FPSb

Next consider the symmetry breaking chain

$$\begin{aligned} \text{SU}(4) \times \text{SU}(3)_A \times \text{SU}(3)_B &\rightarrow \text{SU}(4) \times \text{SU}(2)_L \times \frac{\text{SU}(2)_R \times \text{U}(1)_X}{\mathbb{Z}_2} \\ &\rightarrow \frac{\text{SU}(3)_C \times \text{U}(1)_Y}{\mathbb{Z}_3} \times \text{SU}(2)_W, \end{aligned}$$

where

$$\text{SU}(2)_L \subset \text{SU}(3)_A,$$

and

$$\text{SU}(2)_R \subset \text{SU}(3)_B.$$

If we let

$$U = \underbrace{\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{SU}(3)_B},$$

be the generator of $U(1)_X$, then we can write the hypercharge as

$$Y = \underbrace{\begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{SU(4)} + \underbrace{\begin{pmatrix} \frac{1}{4} & 0 \\ 0 & -\frac{1}{4} \end{pmatrix}}_{SU(2)_R} - \frac{U}{4} = \underbrace{\begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{SU(4)} + \underbrace{\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}}_{SU(3)_B}.$$

This model has similarities with the previous models (both flipped I and flipped II), but the embedding of the intermediate group is different, so the difference between this model and the previous two models lies in the relative scale of the Higgs VEVs.

The relevant differences lie in the nontrivial $SU(4)$ irreducible representations displayed below:

$(4, 2, 1)_{0FPSB}$	$(\bar{4}, 1, 2)_{1FPSB}$	$(4, 1, 1)_{0FPSB}$	$(\bar{4}, 1, 1)_{-2FPSB}$
$\begin{pmatrix} (3, 2)_{\frac{1}{6}} & q \\ (1, 2)_{-\frac{1}{2}} & l \end{pmatrix}$	$\begin{pmatrix} (\bar{3}, 1)_{-\frac{1}{6}} \\ (\bar{3}, 1)_{-\frac{2}{3}} & u^c \\ (1, 1)_{\frac{1}{2}} \\ (1, 1)_0 & \nu^c \end{pmatrix}$	$\begin{pmatrix} (3, 1)_{\frac{1}{6}} \\ (1, 1)_{-\frac{1}{2}} \end{pmatrix}$	$\begin{pmatrix} (\bar{3}, 1)_{\frac{1}{3}} & d^c \\ (1, 1)_1 & e^c \end{pmatrix}$

The up-type electroweak Higgs doublet is contained within $(1, 2, 2)_{-1HFPSB}$ and the down-type Higgs within $(1, 2, 1)_{2FPSB}$, which is the reverse of the case with FPSA.

7.1 Direct breaking

It is also certainly possible that all the GUT Higgs VEVs have the same order of magnitude. In that case, it is more appropriate to say that 433 breaks down directly to the SM gauge group.

8. Trinification

Finally, we return to the trinification case where we will explore the variant of the model with SSB

$$\begin{aligned} SU(4) \times SU(3)_A \times SU(3)_B &\rightarrow SU(3)_C \times SU(3)_L \times SU(3)_R \\ &\rightarrow SU(3)_C \times \frac{SU(2)_W \times U(1)_Y}{\mathbb{Z}_2}. \end{aligned}$$

The \mathbb{Z}_2 modding means that the electric charge is quantized in multiples of $1/3$ [25]. Monopoles form at the TR breaking scale. We choose the fermion content $3[(4, \bar{3}, 1) \oplus (\bar{4}, 1, 3) \oplus (1, 3, \bar{3}) \oplus (1, 3, 1) \oplus (1, 1, \bar{3})]$.

8.1 Model TRa

The first model has

$$\begin{aligned} SU(3)_C &\subset SU(4), \\ SU(2)_W &\subset SU(3)_L = SU(3)_A, \end{aligned}$$

and

$$SU(3)_R = SU(3)_B,$$

with hypercharge

$$Y_{\text{TRa}} = \underbrace{\begin{pmatrix} -\frac{1}{6} & 0 & 0 \\ 0 & -\frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}}_{SU(3)_A} + \underbrace{\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}}_{SU(3)_B}.$$

The decompositions of the representations under TR are

$$\begin{aligned} (4, \bar{3}, 1)_{433} &\rightarrow (3, \bar{3}, 1)_{\text{TR}} \oplus (1, \bar{3}, 1)_{\text{TR}}, \\ (\bar{4}, 1, 3)_{433} &\rightarrow (\bar{3}, 1, 3)_{\text{TR}} \oplus (1, 1, 3)_{\text{TR}}, \\ (1, 3, \bar{3})_{433} &\rightarrow (1, 3, \bar{3})_{\text{TR}}, \\ (1, 3, 1)_{433} &\rightarrow (1, 3, 1)_{\text{TR}}, \end{aligned}$$

and

$$(1, 1, \bar{3})_{433} \rightarrow (1, 1, \bar{3})_{\text{TR}}.$$

8.1.1 Yukawa couplings

The fractionally charged fermions are located within the TR-fundamental fermions fields but not the TR-bifundamental fermions fields. So, we have to pair up the TR-fundamentals with TR-fundamentals. This can be done with a 433 breaking term

$$\langle (\bar{4}, 1, 1)_{H433} \rangle (4, \bar{3}, 1)_{433} (1, 3, 1)_{433}$$

which takes care of $(1, \bar{3}, 1)_{\text{TR}}$ and $(1, 3, 1)_{\text{TR}}$. If we wish to be exhaustive, we should not overlook the possibility of using a VEV for an $SU(3)_A$ -octet

$$\langle (\bar{4}, 8, 1)_{H433} \rangle (4, \bar{3}, 1)_{433} (1, 3, 1)_{433}.$$

This Higgs field can only acquire a VEV at the TR breaking scale, with the corresponding pairing

$$\langle (1, 8, 1)_{HTR} \rangle (1, \bar{3}, 1)_{\text{TR}} (1, 3, 1)_{\text{TR}},$$

which gives a different mass relation between the exotic fermions from the previous choice.

Similarly, we can have either

$$\langle (4, 1, 1)_{H433} \rangle (\bar{4}, 1, 3)_{433} (1, 1, \bar{3})_{433},$$

or

$$\langle (4, 1, 8)_{H433} \rangle (\bar{4}, 1, 3)_{433} (1, 1, \bar{3})_{433}$$

with a different mass relation in each case.

In trinification, we need more than one $(1, 3, \bar{3})_{HTR}$ Higgs field. The other Yukawa couplings terms are

$$\langle (1, 3, \bar{3})_{H433} \rangle (4, \bar{3}, 1)_{433} (\bar{4}, 1, 3)_{433},$$

($(15, 3, \bar{3})_{H433}$ is also possible, but unnecessary since we already have $(1, 3, \bar{3})_{H433}$'s), and

$$\langle (1, 3, \bar{3})_{H433} \rangle (1, 3, \bar{3})_{433} (1, 3, \bar{3})_{433}.$$

This is the only choice which pairs up the exotic leptons. However, it does not cause the SM-neutral fermions to pair up [26]. A $(1, \bar{6}, \bar{3})_{H433}$ leads to antisymmetric mass matrix, but a $(1, \bar{6}, 6)_{H433}$ can be responsible for giving large symmetric masses to the remaining exotics.

8.2 Model TRb

Our second trinification model has

$$\begin{aligned} \text{SU}(3)_C &= \text{SU}(3)_A, \\ \text{SU}(2)_W &\subset \text{SU}(3)_L = \text{SU}(3)_B, \end{aligned}$$

and

$$\text{SU}(3)_R \subset \text{SU}(4),$$

with hypercharge

$$Y_{\text{TRb}} = \underbrace{\begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{\text{SU}(4)} + \underbrace{\begin{pmatrix} -\frac{1}{6} & 0 & 0 \\ 0 & -\frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}}_{\text{SU}(3)_B}.$$

8.2.1 Yukawa couplings

The appropriate Yukawa couplings terms are

$$\begin{aligned} &\langle (\bar{4}, 1, 1)_{H433} \rangle (4, \bar{3}, 1)_{433} (1, 3, 1)_{433}, \\ &\langle (4, 1, 1)_{H433} \rangle (\bar{4}, 1, 3)_{433} (1, 1, \bar{3})_{433}, \\ &\langle (\bar{4}, 1, 3)_{H433} \rangle (4, \bar{3}, 1)_{433} (1, 3, \bar{3})_{433}, \\ &\langle (6, 1, 3)_{H433} \rangle (\bar{4}, 1, 3)_{433} (\bar{4}, 1, 3)_{433}. \end{aligned}$$

As before, the SM-singlet fermions may be paired using a term of the form

$$\langle (10, 1, \bar{6})_{H433} \rangle (\bar{4}, 1, 3)_{433} (\bar{4}, 1, 3)_{433}.$$

8.3 Model TRb'

The third model has some similarities with Model TRa. Here we have

$$\begin{aligned} \text{SU}(3)_C &= \text{SU}(3)_B, \\ \text{SU}(2)_W &\subset \text{SU}(3)_L \subset \text{SU}(4), \end{aligned}$$

and

$$\mathrm{SU}(3)_R = \mathrm{SU}(3)_A,$$

with hypercharge

$$Y_{TRb'} = \underbrace{\begin{pmatrix} -\frac{1}{6} & 0 & 0 & 0 \\ 0 & -\frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{\mathrm{SU}(4)} + \underbrace{\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix}}_{\mathrm{SU}(3)_A}.$$

8.3.1 Yukawa couplings

The necessary Yukawa couplings terms are

$$\begin{aligned} &\langle (\bar{4}, 1, 1)_{H433} \rangle (4, \bar{3}, 1)_{433} (1, \bar{3}, 1)_{433}, \\ &\langle (4, 1, 1)_{H433} \rangle (\bar{4}, 1, \bar{3})_{433} (1, 1, \bar{3})_{433}, \\ &\langle (4, \bar{3}, 1)_{H433} \rangle (\bar{4}, 1, \bar{3})_{433} (1, \bar{3}, \bar{3})_{433}, \\ &\langle (6, \bar{3}, 1)_{H433} \rangle (4, \bar{3}, 1)_{433} (4, \bar{3}, 1)_{433}, \end{aligned}$$

and the SM-singlet fermions may be paired up via

$$\langle (\bar{10}, 6, 1)_{H433} \rangle (4, \bar{3}, 1)_{433} (4, \bar{3}, 1)_{433}.$$

For the remaining models in our classification, we have not been able to find a suitable choice of additional “exotic” fermion fields to add to make **all** the exotic fermions vectorlike at the GUT scale. While this certainly does not mean that such a combination is not possible, it probably means that any such combination would have to be fairly complicated.

9. Discussion

Perhaps the most unusual and interesting new models found here are those where some of the extra fermions have X charges. We can generate other models similar to the X model by extending the gauge group of the PS or TR model. To see this let one of the $\mathrm{SU}(N)$ s be extended to $\mathrm{SU}(N+2)$ in a way that the families have no charge under the new diagonal generator $X' = \mathit{diag}(0, 0, \dots, 0, 1, -1)$ of $\mathrm{SU}(N+2)$. For example, let G_{TR} be replaced by $\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(5)$, where we then break this symmetry to $G_{\mathrm{TR}} \times \mathrm{U}(1)_{X'}$. Then the TR fermions which must be extended to

$$5(\bar{3}, \bar{3}, 1) + 3(1, \bar{3}, \bar{5}) + 3(\bar{5}, 1, \bar{3}) \quad (9.1)$$

under $G_{\mathrm{TR}} \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(5)$, reduce to

$$5(\bar{3}, \bar{3}, 1)_0 + 3(1, \bar{3}, \bar{3})_0 + 3(\bar{3}, 1, \bar{3})_0 + (1, \bar{3}, 1)_{x'} + (\bar{3}, 1, 1)_{x'} + (1, \bar{3}, 1)_{-x'} + (\bar{3}, 1, 1)_{-x'}. \quad (9.2)$$

This model has some interest in its own right since family unification is required if we only allow bifundamentals (However, problems can arise in this model with respect to giving heavy masses to the exotics.). As planned, the three families have no X' charge. Therefore,

the breaking of $U(1)_{X'}$ is not tied to the charge operator for standard families, nor is the X' charge *a priori* quantized for the extra fermions. The X model itself is somewhat more interesting since it is not one of these extensions. $U(1)_X$ is an integral part of the 334 model and is interwoven into $SU(4) \times SU(3) \times SU(3)$ in a nonfactorizable way. As we have seen, there are a number of choices for x that can be used to flip the X model. Likewise, this can be done for X' models.

The spontaneous symmetry breaking of $SU(4) \times SU(3) \times SU(3)$ to models that contain the TR or PS model is straightforward. A Higgs multiplet $(4, 1, 1)_H + h.c.$ can be used to break directly to the $SU^3(3)$ of the TR model. Further breaking proceeds as in the TR model. Likewise a $(1, 3, 1)_H + h.c.$ and a $(1, 1, 3)_H + h.c.$ can be used to break $SU(4) \times SU(3) \times SU(3)$ to the PS group $SU(4) \times SU(2) \times SU(2)$. Further breaking proceeds as in the PS model. Hence, we only need to concern ourselves with the 334 that is not equivalent to the TR or PS models. The SSB to any of the models listed can always be achieved, given sufficient freedom in the Higgs sector. It would be interesting if we could derive any of the 334 type models from an orbifolded *AdS/CFT* theory, as this would strongly restrict both the fermion and scalar content of the theory, i.e., adjoints and bifundamentals only.

In the PS and TR models, several symmetry breaking scales can be associated with magnetic monopoles. In this context it is important that the gauge boson mediated proton decay is absent in these models, but proton decay can still proceed through Higgs (Higgsino if the model is supersymmetrized) exchange, as well as via higher dimension operators. It is relatively straightforward to construct models based on G_{PS} and G_{TR} where proton decay is forbidden, as a consequence, say of an ‘accidental’ baryon number symmetry. This allows the possibility that G_{PS} and G_{TR} could be broken at scales far below the conventional grand unification scale $M_{GUT} \sim 10^{16} GeV$. An example based on D-branes in Type I string theory was provided for the G_{PS} symmetry [27], where with the standard embedding of $SU(3)_C \times SU(2) \times U(1)$, the symmetry breaking scale of G_{PS} becomes $M_{PS} \sim 10^{12} - 10^{13} GeV$, with the corresponding string scale $\gtrsim M_{PS}$. Thus, monopoles with mass $\sim 10^{13} - 10^{14} GeV$ are expected in this class of models. An even more suggestive result is given by the PS type model based on *CFT* obtained from orbifolded type *IIB* strings [28, 29]. Here the unification is in the 100 *TeV* range, and other intriguing phenomenology possibilities appear, e.g., $\sin^2\theta_W$ can be approximately .23, etc. [30]. Analogous considerations should apply to the trinification scheme and, by extension, to the gauge symmetry of special interest here $SU(4) \times SU(3) \times SU(3)$. Assuming for instance that a 334 model could be derived from an orbifolded $AdS \times S^5$, the multiply charged monopoles of the theory will have mass as small as $M \sim 10^7 GeV$, which is in the preferred range of interest if they are to be candidates for high energy cosmic ray primaries [31, 32]; a more detailed study of mass scales would require an RG analysis for each model. We expect the 334-model to have a similar unification scale with resulting exotic (fractionally charged) leptons and/or hadrons, and we expect their masses to be near this unification scale, so they are also of interest as dark matter candidates [33, 34].

Given that monopoles of mass of order $\sim 10^{13} - 10^{14} GeV$ (or perhaps even much lighter) can arise in realistic models, it is important to ask: Can these primordial monopoles survive inflation? A non-supersymmetric inflationary scenario which dilutes but does not

completely wash away intermediate mass monopoles was developed in ref [35]. The D-brane scenario discussed above gives rise to non-supersymmetric $SU(4)_C \times SU(2) \times SU(2)$, so the discussion in ref [35] may be relevant. The monopole flux can be reduced to close to the Parker bound $10^{-16} cm^{-2} s^{-1} sr^{-1}$. In the orbifolded scheme, the SSB scale where the monopoles get their masses can be below the inflation scale. Hence, the monopoles can exist in interesting densities (near the Parker bound) depending on details of the SSB phase transitions. For the supersymmetric case, dilution of monopoles can be achieved by thermal inflation [36, 37] followed by entropy production. A scenario in which thermal inflation is associated with the breaking of the U(1) axion symmetry was recently developed in ref [38]. Current experimental limits on the relativistic monopole flux in cosmic rays have been pushed below the Parker bound [39–42], while direct searches have also constrained the monopole mass and cross section [44, 43]. See also the discussion in [45].

In summary, the $SU(4) \times SU(3) \times SU(3)$ models we are advocating provides a natural family unification while avoiding proton decay and giving rise to both (exotic) fractionally charged color singlets and corresponding multiply charged magnetic monopoles [46] with densities compatible with the Parker bound, and with masses perhaps as light as $\sim 10^7$ GeV, (Note that in SU(5) the lightest monopole has mass of $\sim 10^{17}$ GeV, and carries one unit of magnetic charge [47].) The exotic states are heavy (greater than a few TeV), but may be in the range explored by accelerators in the next decade.

Acknowledgments

TWK thanks the Bartol Research Institute at the University of Delaware and the Aspen Center for Physics for hospitality while this work was in progress. This work was supported by the US Department of Energy under Grants No. DE-FG05-85ER40226 (TWK) and by DE-FG02-91ER40626 (QS).

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- [9] Recall that three standard families of chiral fermions at the EW breaking scale are in the representations: $3[(3, 2)_{\frac{1}{6}} \oplus (\bar{3}, 1)_{-\frac{2}{3}} \oplus (\bar{3}, 1)_{\frac{1}{3}} \oplus (1, 2)_{-\frac{1}{2}} \oplus (1, 1)_1]$. If we include right handed neutrino, they are in neutral singlets $(1, 1)_0$.
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- [16] It is standard practice to normalize all U(1) generators Λ such that $Tr[\Lambda^2] = 2$ but we usually find it more convenient to drop the overall normalization factors as will be either stated or clear from context.
- [17] Technically speaking, we should not be using the vacuum since hypercharge is spontaneously broken in the vacuum. We should be using some thermal state in equilibrium at a temperature above the electroweak phase transition.
- [18] This can be traced to the fact that we are breaking down from $SU(4) \times SU(2)_L \times SU(2)_R$ instead of $\frac{SU(4) \times SU(2)_L \times SU(2)_R}{\mathbb{Z}_2}$
- [19] In the rest of this draft, we will append a subscript to each representation indicating the Lie group it is a representation of to reduce any potential ambiguities that may arise.
- [20] This might actually allow us to reduce the number of Higgs fields.
- [21] A subscript of H indicates that it is a Higgs field. The absence of such a subscript indicates that it is a fermion field.
- [22] Let's say an SU(3) symmetry breaks down to SU(2) and any Higgs VEV has to be invariant under SU(2). Let's also say that we have two SU(3)-triplet fermions. The SU(3)-triplets decompose into an SU(2)-doublet and an SU(2)-singlet. Suppose we wish to pair up the

SU(2)-doublets using a Yukawa coupling to a Higgs field. Then a $\bar{6}_{HSU(3)}$ Higgs does not work because $\langle \bar{6}_{HSU(3)} \rangle_{SU(3)} 3_{SU(3)} 3_{SU(3)}$ only pairs up the SU(2)-singlets but not the SU(2)-doublets. So we have to go for the $\langle 3_{HSU(3)} \rangle_{SU(3)} 3_{SU(3)} 3_{SU(3)}$ coupling instead, which only pairs up the SU(2)-doublets but not the SU(2)-singlets. Conversely, if we only wish to pair up the SU(2)-singlets but not the SU(2)-doublets, we have to choose $\bar{6}_{HSU(3)}$ and if we wish to pair up both the SU(2)-doublets and the SU(2)-singlets, we have to include both Higgs fields.

- [23] This will be abbreviated as FPSA.
- [24] There could be some additional symmetries preventing such couplings, or maybe it is the adjoint $15_{SU(4)}$ which is responsible for the symmetry breaking. However, the further flipping doesn't really lead to any problems, so it is not necessary to have such a principle in the first place.
- [25] Whether the scale happens to be above or below the QCD scale is irrelevant here since there are electrically neutral colored particles. This is unlike the minimal SM where even though the electric charge is quantized in multiples of 1/3 at distance scales smaller than the QCD scale, only integer charges are present at larger scales.
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